Synopsis of Proposed Research Work Entitled

"Some Studies in Non-Linear Differential Equations using Iteration Method with its Applications"

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Ph.D. Synopsis

"Some Studies in Non-Linear Differential Equations using Iteration Method with its Applications"

Abstract of proposed research work:

1. Introduction :

The nonlinear equations play a vital role in modelling natural phenomena. In general, it is not possible to provide exact solution to such equations. In this case one has to find either numerical solutions or approximate analytical solutions.The numerical solutions are obtained by discretizing the solution interval. The step size should be chosen sufficiently small to avoid errors. Further, the values of all parameters in the equation should be known. The numerical methods usually provide solutionon large intervals.If one is interested in analytical solutions of nonlinear equations then some iterativeor decomposition methods are very useful. The solutions provided by such methods areoften in the form of a series. If the number of this series terms is sufficiently large thenthe error in the solution is negligible. In this case, we don't need to put values of all parameters. However, the solutions are usually local unlike numerical solutions. We take a review of some iterative methods for solving nonlinear equations.

a) Variational Iteration Method : The variational iteration method (VIM) is proposed by J. H. He by utilizing thetechniques of calculus of variations. VIM has been used to solve nonlinear problems suchas ODEs, PDEs Burger's equation, Helmholtz equation, Schrodingerequation, and Klein-Gordon equation.The method is used to solve the following form of nonlinear differential equation,

$$L\left(u\left(x\right)\right) + N\left(u\left(x\right)\right) = g\left(x\right). \quad (1)$$

According to the variational iteration method, we can construct a correction functional for Eq. (1) as,

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda \left(Lu_n(t) + Nu_n(t) - g(t) \right) dt \qquad (2)$$

where λ is a general Lagrange multiplier which can be identified optimally via the variational theory. Here, we apply restricted variations to nonlinear term $N(u)$ and determine the multiplier. The term $\tilde{u}_n$ considered as a restricted variation where $\delta\tilde{u}_n = 0$.

b) Three iterative methods have been implemented to solve several second order nonlinear ODEs that arising in physics. The proposed iterative methods are **Tamimi-Ansari method (TAM)**, **Daftardar-Jafari method (DJM)** and **Banach contraction method (BCM).** Each method does not require any assumption to deal with nonlinear term. The obtained results are compared numerically with other numerical methods such as the Runge-Kutta 4 (RK4) and Euler methods. In addition, the convergence of the proposed methods is given based on the Banach fixed point theorem. The results of the maximal error remainder values show that the present methods are effective and reliable.

c) Dhage iteration method This Method provides algorithms for the existence and approximation of the solutions for an initial and a periodic boundary value problem of nonlinear first order ordinary hybrid differential equations with a linear perturbation , the existence and approximation result for a nonlinear initial value problem of second order hybrid functional differential equations of neutral type via construction of an algorithm. The main results rely on the Dhage iteration method embodied in a recent hybrid fixed point principle of Dhage .

2 Origin of the research Problem:

In the physical world nothing is permanent except changes. These changes take place not arbitrarily but occur according to the general laws of nature. The quest of human being, from immemorial time, was to know these laws and the surrounding world. The best way is to know the interrelated changing quantities in the laws of nature. In the language of mathematics these quantities are called variables. The study of these variables leads to differential equation that transforms information from the physical world to mathematical equation thus the differential equation play the role of models. This is the reason that we find number of applications of differential equation in physics, chemistry, biology, astronomy, engineering economics and other fields of applied sciences. As regard to origin of differential equation, the study began soon after invention of differential and integral calculus. It will not be an exaggeration if we say that Newton's was first to solve differential equation in 1676.

Analytical method for DE

$$f\left(x, y, y', \ldots\right) = 0$$

give exact solution in explicit or implicit form in terms of known function such as polynomials, trigonometric or exponential function, however such methods are limited and are applicable to any such special type of DE. For DE which is not solvable by analytical method to make the system workable we resort to numerical approximation. It has rule or algorithm and the set of solution is derived by iterations. We say that solution arrived at is by recursion. Elementary methods of numerical solution of ode include Euler's method, Heurs method, Runge-Kutta method, Milne's method, Adams method etc.

3 Review of Literature and Development in the subject (Previous work done in the related area):

Non-linear problems of real-life problem are still difficult to solve theoretically. A broad class of numerical solutions methods were used to handle these problems by Wang (1998); Jeffery and Mohamad (1991); Wadati (1972) etc. The Klein-Gordan DE is one of the most important mathematical models is quantum mechanics (whitham,1974;Zauderer 1983).

Variational iteration method (VIM), a powerful method was first envisioned by He (1998) VIM has successfully been applied to many situations. He (1998) solved the classical Blasius' Equation using VIM,(2000) solved autonomous ordinary differential system using VIM and (2006) solved strongly non-linear equations using VIM. Soliman (2005) applied VIM to solve the Kdv-Berger's and Laxs Seventh- order Kdv equation. VIM was employed for solving Nonlinear coagulation problem with mars loss by Abulwataetal.(2005) Momanietal (2006) applied VIM to the Helmholtz equation, with the passage of time same modifications in Hes VIM has been introduced by various authors. Abbasbandy (2007), made the coupling of Adomian's polynomials with the correction functional (VIMAP) of the VIM and applied this relative versionfor solving Riccati differential and Klein Garden Equation. In a later work, Noor and Mohyud-Din (2008) explained this concept for solving various singular and non singular boundary value problems. Recently,Ghorbani et al.[2007,2009] introduced He's Polynomialsby splitting the non linear term and also proved that He's polynomials
are fully compatible with Adomian's polynomial but are easier to calculate and more user friendly.

A new iterative method has been presented in 2011 by Temimi and Ansari (TAM) (Temimi and Ansari, 2011) for solving nonlinear problems. The TAM was inspired from the homotopy analysis method (HAM) (Liao and Chwang, 1998), and used to solve several ODEs (Temimi and Ansari, 2015), PDEs and KdV equations (Ehsaniet al., 2013); differential algebraic equations (DAEs) (AL-Jawaryand Hatif, 2017); Duffing equations (AL-Jawary and Al-Razaq,2016); some chemical problems (AL-Jawary and Raham, 2017), thin film flow problem (AL-Jawary, 2017) and Fokker-Planck's equations (AL-Jawary et al., 2017). The other proposed method has been proposed in 2006 by Daftardar-Gejji and Jafari, (DJM) (Daftardar-Gejji and Jafari, 2006) to solve nonlinear equations. Also, this method has been used to solve different equations such as fractional differential equations (Daftardar-Gejji and Bhalekar, 2008), partial differential equations (Bhalekar and Daftardar-Gejji, 2008); Volterra integro-differential equations and some applications for the Lane-Emden equations(AL-Jawary and AL-Qaissy, 2015), evolution equations (Bhalekar and Daftardar-Gejji, 2010). This method presented a proper solution which converges to the exact solution ''if such solution exists" through successive approximations. The other iterative method depends on the Banach contraction principle (BCP) (Daftardar-Gejji and Bhalekar, 2009) where it isanother iterative method considered as the main source of the metric fixed point theory. The Banach contraction principle also known to be Banach's fixed point theorem (BFPT) has been used to solve various kinds of differential and integral equations (Joshi and Bose, 1985). the TAM, DJM, and BCM will be implemented to solve the nonlinear second order ODEs that arising in physics to get an approximate solutions. These solutions will be compared numerically with another results obtained by the Runge-Kutta and Euler methods to show the validity and the efficiency of the proposed

iterative methods. The differential equations involving the history of the dynamic systems are called functional differential equations and the differential equations involving the derivative of history function are called neutral functional differential equations. It has been recognized long back the importance of such problems in the theory of differential equations. Since then, several classes of nonlinear functional differential equations of neutral type have been discussed in the literature for different qualitative properties of the solutions (see Ntouyas [21] and the references therein)., the study of a special class of functional differential equations involving maxima is initiated by Dhage [22], Dhage and Otrocol [22] and Dhage and Dhage [23] via a new Dhage iteration method and established the existence and approximation results along with algorithm of the solutions.

4 Objectives of Research / Proposed Hypothesis:

The main objectives of this study are

I. To study various methods for solving differential equations.
II. To study various Iteration methods for solving non-linear differential equations.
III. To study existing methods and their applications.
IV. To formulate new technique for solving non linear differential equation and compare it with existing methods.
V. To study applications related to material science, engineering and technology etc.

5 Methodology to be adopted:

I. Initially, for the review purpose relevant research papers, review articles, books, journals etc. will be collectedthrough the internet and various institutions.
II. To participate in conferences, seminars, workshops.
III. To obtain an alternative solution of the problem by applying new technique and compare our result with obtained already results.
IV. To use computational software viz. Maple, Mathematica, MathCAD, MATLAB etc.
V. The carried out researchwork will be published in the well reputed journals.

6 Importance of study/ society application:

Interdisciplinary Relevance:

The issue has great importance because it is an interdisciplinary work related with the branch of science and technology like physics, Mechanical Engineering and Civil Engineering etc.

International & National Status:

Lot of work has been carried out by many national and international researchers on differential equations. The works of few amongst them are quoted in the direct and indirect problems. **(**Refer Review of Literature).

Significance of the study:

Differential equation is becoming useful and key tool in the analysis of scientific problems in physics, engineering and its formulation is useful for solving real life problems.

7 Proposed work Plan / Formulation & Structure of Study:

Year wise plan of work and targets to be achieved are as follows.

1. **First Year:**
   - i. To visit various institutes, librariesand using internet to collect the references on the proposed research topic.
   - ii. To study the collected research material.
   - iii. To attend the seminars/workshops on relevant topic.
   - iv. To develop new models on the topic.

2. **Second Year:**
   - i. To attend the seminars/conference/workshops on relevant topic.
   - ii. To formulate new problems onrelevant topic.
   - iii. To study existing and new technique will be applied on these problems.
   - iv. To construct the mathematical models and study their solutions.
   - v. To check the applicability of the proposed method.

3. **Third Year:**
   - i. To study applications of obtained results.
   - ii. To publish the work in reputed journals.
   - iii. To submit the work to the University.

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